Geometry, Quarter 1, Unit 1.1

Introducing Geometric Basics

Overview

Number of instructional days: 14 (1 day = 45–60 minutes)

Content to be learned

- Experiment with transformations of angles in a plane.
- Experiment with transformations of perpendicular lines in a plane.
- Experiment with transformations of parallel lines in a plane.
- Experiment with transformations of line segments in a plane.
- Prove theorems about congruency of vertical angles.
- Prove theorems about congruency of parallel lines.
- Prove theorems about congruency of alternate interior angles.
- Prove theorems about congruency of corresponding angles.
- Prove perpendicular bisector theorem.
- Make formal geometric constructions using a variety of tools and methods.

Mathematical practices to be integrated

Model with mathematics.

- Use constructions with lines and angles.
- Construct transformations of angles, line segments, and parallel and perpendicular lines, on a coordinate plane.

Use appropriate tools strategically.

- Use a protractor to find measures of angles and lines.
- Use paddy paper to transform lines.
- Use a compass to bisect angles.
- Use dynamic geometric software to view angle and line measures.

Attend to precision.

- Determine angle measures accurately using a protractor.
- Illustrate the method to use a compass to determine if lines are parallel.
- Demonstrate algebraically that vertical angles, alternate interior angles, and corresponding angles are congruent.

Essential questions

- What were the findings about transformations of angles after experimenting with them in the coordinate plane?
- What methods were used in geometric constructions?
- How can a transformation be described with a function?
- What is the definition of transformation in terms of angles, perpendicular lines, parallel lines, and line segments?
- What is the precise language to describe a geometric transformation in a plane?

Written Curriculum

Common Core State Standards for Mathematical Content

Congruence	**G-CO**

Experiment with transformations in the plane

G-CO.1 Know precise definitions of angle, ~~circle,~~ perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line~~, and distance around a circular arc.~~

Prove geometric theorems

G-CO.9 Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*

Make geometric constructions

G-CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line*.

Common Core Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

In grade 4, students drew points, lines, line segments, rays, angles, and perpendicular and parallel lines. They identified these elements in two-dimensional figures. They also classified shapes by properties of their lines and angles. Fifth-grade students extended this knowledge to the coordinate plane. In grade 8, students experimented with rigid motion through transformations that preserved distance and angle measure, and they described the effect of dilations on two-dimensional figures using coordinates.

Current Learning

In this unit, students learn precise geometry definitions for terms such as angles, perpendicular lines, parallel lines, and line segments based on the undefined notions of point, line, and distance along a line. Students also represent transformations and describe them as functions. They experiment with transformations in the plane. Students continue to justify and explain the basis of movement in rigid motion of geometric concepts. Students investigate analytic geometry in the coordinate plane.

In this unit, students develop informal and formal proofs of previously learned concepts and apply them as they did in earlier grades. Students also use measurement of line segments as a tool in developing the idea of proof. For example, students prove theorems relating to congruence of vertical angles, transversals and parallel lines, and points on a perpendicular bisector of a line segment. Geometry students learn to write proofs in multiple ways, including narrative paragraphs, flow diagrams, the two-column format, and diagrams without words. According to Appendix A, "Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning."

Future Learning

In algebra 2 and precalculus, students will continue to see transformations on a coordinate plane relating to functions. Understanding that transformations are used in all subsequent mathematical and science based work for students is imperative. Students will be able to use their knowledge and skills with transformations in such fields as management occupations, computer and mathematical occupations, architects, surveyors, cartographers, construction workers, and administrative positions.

Additional Findings

The work in this unit is challenging for students because of the precise language and extensive vocabulary it requires. While students may have seen this unit's vocabulary before, the terms are not used in everyday language and may be particularly difficult for English-language learners. It is important to clarify multiple meanings of words so that students understand that mathematics requires very precise and specific definitions and understanding of the terms. "High school students should learn multiple ways of expressing transformations . . . as well as function notation." (*Principles and Standards for School Mathematics*, p. 43)

Geometry, Quarter 1, Unit 1.2

Proving Geometric Theorems Algebraically

Overview

Number of instructional days: 6 (1 day = 45–60 minutes)

Content to be learned

- Prove simple geometric theorems algebraically.
- Prove simple geometric theorems using coordinates.
- Prove algebraic equations using a two-column proof.
- Prove algebraic equations using a paragraph proof.
- Prove algebraic equations using a flow chart proof.
- Prove algebraic equations using an indirect proof.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Discuss an algebraic proof and predict the outcome.
- Create a table to prove an algebraic equation.

Construct viable arguments and critique the reasoning of others.

- Write a two-column proof using algebra.
- Write a paragraph proof to solve an algebraic equation.
- Illustrate a flow chart proof algebraically.

Model with mathematics.

- Use chart paper to illustrate a flow chart proof.
- Demonstrate a two-column proof on a poster board.

Look for and express regularity in repeated reasoning.

- Use definitions, postulates, and theorems as reasons to show the logical order of solving an algebraic equation in a paragraph proof.

Essential questions

- What are the similarities and differences between indirect and direct proofs?
- How are flow-chart proofs and two-column proofs related?
- Why are paragraph proofs important for students?
- When is it important for students to be able to justify their findings in a proof?

Written Curriculum

Common Core State Standards for Mathematical Content

Expressing Geometric Properties with Equations	**G-GPE**

Use coordinates to prove simple geometric theorems algebraically

G-GPE.4 Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2).*

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

In sixth grade, students learned how to set up one-variable equations and inequalities and then solve them. In previous grades, students informally used reasoning to draw conclusions and to make and justify conjectures and arguments in both written and oral form. In grade 8, students used ideas about distance and angles and how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students showed that the sum of the angles in a triangle is the angle formed by a straight line and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students also applied theorems, such as the Pythagorean Theorem, to solve problems.

Current Learning

In this unit, students develop informal and formal proofs of algebraic concepts and apply them as they did in earlier grades. Students also use measurement of line segments as a tool in developing the idea of proof. For example, students prove theorems relating to congruence of vertical angles, transversals and parallel lines, and points on a perpendicular bisector of a line segment. Geometry students learn to write proofs in multiple ways, including narrative paragraphs, flow diagrams, the two-column format, and diagrams without words.

Future Learning

Students will continue to need the ability to reason and justify through proof, both formally and informally. Later in algebra 2 and precalculus, as well as in real-world situations, students will continue to use logical reasoning in all problem situations. This will particularly apply to the field of law because lawyers take all the facts and use deductive logic to figure out ways to defend a case.

Additional Findings

Using logical thinking to prove theorems is challenging for students because it requires them to think in a new and precise way about situations presented to them. Many students have difficulty with formulating justifications for their conclusions, especially in mathematics. “Proof is a form of justification, but not all justifications are proofs.” (*Adding It Up*, p. 132)

Proofs are challenging for both students and teachers, since “conjectures are not the same as proofs. Finding precise descriptions of conditions for the first step is important.” Students are not accustomed to justifying their reasoning using formalized methods of proof. (*Principles and Standards for School Mathematics*, p. 311)

Another challenge is that as students progress through levels of learning and understanding in mathematics, “each level has its own language and way of thinking; teachers unaware of this hierarchy of language and concepts can easily misinterpret students’ understanding of geometric ideas. At Level 4, students can establish theorems within an axiomatic system.” A misconception to be overcome is that students have difficulty understanding the difference between an explanation and a justification. (*A Research Companion to Principles and Standards for School Mathematics*, p. 152)

Geometry, Quarter 1, Unit 1.3

Proving Triangles are Congruent

Overview

Number of instructional days: 19 (1 day = 45–60 minutes)

Content to be learned

- Prove triangle congruency in term of rigid motion.
- Prove triangle congruency by Angle-Side-Angle Postulate.
- Prove triangle congruency by Side-Side-Side Postulates.
- Prove triangle congruency by Side-Angle-Side Postulate.
- Prove the Triangle Sum Theorem.
- Prove Base Angles Theorem of a triangle.
- Prove the slope criteria for parallel and perpendicular lines to solve geometric problems using coordinates.
- Prove the slope criteria to solve geometric problems.
- Find a point on a directed line segment between two given points that partitions the segment in a given segment.
- Compute the perimeter and area of triangles and rectangles using the distance formula.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Draw triangles to predict angle and side congruency.
- Graph linear lines to find the slope.

Reason abstractly and quantitatively.

- Discuss the postulates needed to determine triangle congruency.
- Use the distance formula to establish triangle and rectangle measures.

Construct viable arguments and critique the reasoning of others.

- Graph linear equations to determine the point of intersection.
- Write a paragraph proof that signifies triangle congruency.

Look for and make use of structure.

- Observe the similarities between the Side-Side-Side congruence postulate and the Side-Angle-Side congruence postulate.
- Glance at triangle rigid motion to conclude congruency.

Essential questions

- What criteria would you use to determine which congruency postulate to use?
- How are all the triangle congruencies postulates similar and different?
- How does rigid motion relate to triangle congruence?
- How can you prove two triangles are congruent?

Written Curriculum

Common Core State Standards for Mathematical Content

Congruence	**G-CO**

Understand congruence in terms of rigid motions

G-CO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions

G-CO.10 Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent~~; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.~~*

Expressing Geometric Properties with Equations	**G-GPE**

Use coordinates to prove simple geometric theorems algebraically

G-GPE.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

G-GPE.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

G-GPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.★

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches..

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments..

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

In kindergarten, students were introduced to geometric shapes. In first through third grade, students reason with shapes and their attributes. Students were introduced to angles and measuring angles in grade 4. In middle school, students used triangle relationships to solve problems, including determining when three angles or sides create a unique triangle, more than one triangle, or no triangle.

In grade 7, students reasoned about relationships among two-dimensional figures using scale drawings and informal geometric constructions. They described geometric figures and the relations between them and they drew geometric shapes with given conditions. In grade 8, understanding congruence was a major cluster. Students worked with congruence and similarity using physical models, transparencies, and geometry software. In their previous course work with geometric concepts, students have developed fluency in a strong set of measurement skills.

Current Learning

Conceptual understanding of triangle congruence is a fluency recommendation for students. Understanding triangle congruence is a foundation for continued success in geometry. In this course, students formalize triangle relations they learned in middle school into theorems. Geometry students establish congruence between two triangles using rigid motion. Students investigate and formally prove, through a variety of methods, triangle congruence using the following postulates and theorems: SAS, ASA, and SSS. They extend their understanding to properties of isosceles and equilateral triangles and prove theorems about triangles, including that the measures of interior angles of a triangle add up to 180°, the base angles of isosceles triangles are congruent, the segment joining the midpoints of two sides of a triangle is parallel to the third side and half its length, and the medians of a triangle meet at a point.

Future Learning

This unit will lay a foundation for geometric applications in subsequent mathematics courses. A thorough knowledge of triangle congruence will prepare students to solve problems from physical situations using trigonometry, including the use of the Law of Sines, the Law of Cosines, and area formulas. Students will also encounter coordinate transformations using matrices in precalculus. In the future, students will be able to use their knowledge and skills of triangle congruency in careers such as architecture, engineering, industrial engineering, and as a surveyor.

Additional Findings

There are no additional findings for this unit.